

# THE HYBRID PARAMETRIC AMPLIFIER

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## ABSTRACT

This paper describes in theory and in practice a non-degenerate non-oscillatory Parametric Amplifier called the Hybrid Parametric Amplifier. Practical measures confirm the theory and, in particular, at 30 MHz a noise temperature of 18°K coupled with a gain of 6dB is obtained.

### Introduction

The conventional Paramp consists of two resonant circuits as a consequence of which the amplifier can oscillate if the energy transferred to the signal circuit is more than the energy loss. Hence, the resonant Paramp requires an accurately stabilised pump source.

The Travelling Wave Paramp consists of an artificial transmission line, containing varactors which propagates the signal, idling and pump energies as travelling waves. This introduces a phase requirement between the signal idling and pump energy at each varactor which is difficult to maintain. Hence, most modern Paramps are resonant.

The Paramp described below combines the two techniques and is called the Hybrid Paramp. (References 1,2,3).

### The Hybrid Paramp

#### The Principle of the Hybrid Paramp

The Hybrid Paramp consists of an artificial transmission line at the signal frequency, with series inductances and shunt varactor elements. Associated with each varactor there is a separate resonant idler circuit. Pump power is fed to each varactor independently. Fig. 1 shows a single T-section of the Hybrid Parametric Amplifier.

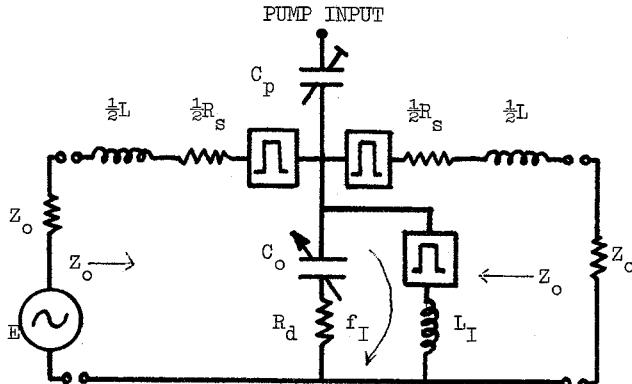


Fig. 1 Single T-section of Hybrid Parametric Amplifier

The varactor  $C_o$ ,  $R_d$  is resonated by  $L_I$  at the idling frequency  $f_I$ , the series arms of the T-section are inductors  $\frac{1}{2}L$ ,  $\frac{1}{2}R_s$  and filters are provided to confine energy to the appropriate part of the circuit. Pump power is fed to the varactor through a capacitor  $C_p$ . The T-section is terminated in its character-

istic impedance,  $Z_o$ , at both ends.

#### Gain Expression

Calculation of the expression for  $Z_o$  for one T-section shows that:

$$Z_o = \sqrt{\frac{L}{C_o} \left( 1 - \frac{\omega_s^2 L C_o}{4} \right)} \quad (1)$$

where  $\omega_s$  is the signal frequency propagating down the transmission line. The current  $I_L$  flowing through the load is given by

$$I_L^2 = \frac{\left[ (R_d - R_n)^2 + \frac{1}{\omega_s^2 C_o^2} \right] E^2}{\left[ (Z_o + \frac{1}{2}R_s)^2 + \left( \frac{\omega_s L}{2} \right)^2 \right] \left[ (Z_o + \frac{1}{2}R_s + 2R_d - 2R_n)^2 + \left( \frac{4}{\omega_s^2 C_o^2} \right) \left( 1 - \frac{\omega_s^2 L C_o}{4} \right)^2 \right]} \quad (2)$$

where  $R_n$  is  $\gamma^2 / (C_o^2 \omega_s \omega_I R_d)$ ,  $\omega_I$  is the idling frequency and  $E$  the generator e.m.f.

Note that when  $2R_n$  equals  $Z_o + \frac{1}{2}R_s + 2R_d$  the second denominator term does not vanish (as it does in the resonant Paramp) and the current through the load therefore has a finite maximum value. Thus the amplifier does not oscillate. The maximum available gain  $G_{av,max}$  can be calculated as:

$$G_{av,max} = \frac{1 + g(1 - g)}{(1 + \frac{g}{1 - g})(1 - g)^2} \quad (3)$$

where we have written  $\omega_s^2 L C_o / 4$  as  $g$ . The above expression simplifies to

$$G_{av,max} = g + \frac{1}{(1 - g)} \quad (4)$$

Variation of gain with  $g$  is shown in Fig. 2 below. Also shown (Fig. 3) is the variation of gain with pumping level ( $\gamma$ ) for a particular  $g$  selected to give a maximum gain of 6dB. This illustrates the theoretical dependence of gain of pump level showing the zero slope at the optimum gain.

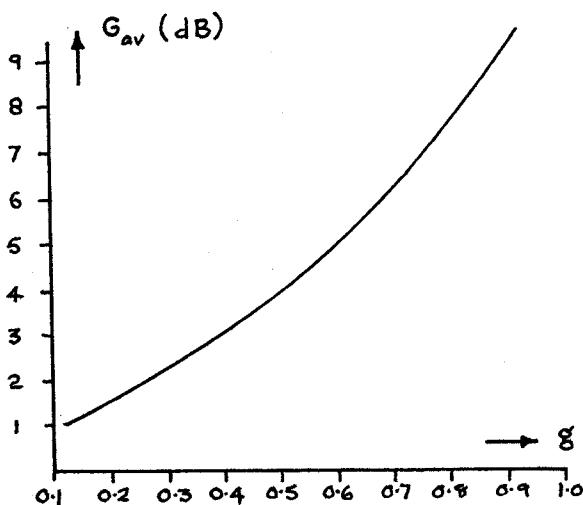


Fig. 2 Dependence of Available Gain on  $g$

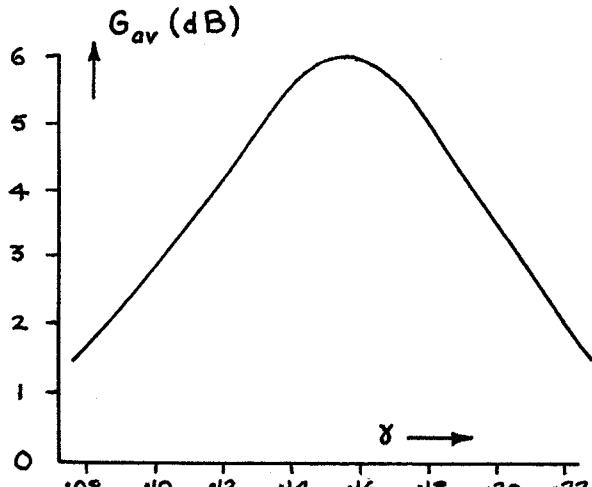


Fig. 3 Theoretical variation of gain as a function of  $\gamma$

In practice, once a varactor is chosen, the gain is determined by selecting the appropriate value of  $L$  for the series arm of the T-section.

#### Noise Figure Expression

To calculate the noise figure of a single T-section we have to determine the noise dissipated in the load by the following thermal sources:

- 1) the source impedance ( $Z_o$ )
- 2) the series loss associated with the first half of the inductance  $\frac{1}{2}L$
- 3) the series loss associated with the second half of the inductance  $\frac{1}{2}L$
- 4) the varactor loss,  $R_d$ , in the signal circuit
- 5) the varactor loss,  $R_d$ , in the idler circuit

Conventional analysis for these sources produces the following expressions for the dissipated noise powers

$$k T_o B (g + \frac{1}{1-g}) \quad (5)$$

$$\frac{k T_o B}{q} (g + \frac{1}{1-g}) \text{ where } q = Z_o / 2R_s \quad (6)$$

$$\frac{k T_o B}{q} \left[ g + \left( \frac{1-2g}{1-g} \right)^2 \right] \quad (7)$$

$$\frac{k T_o B}{p} \left( \frac{2g}{1-g} \right) \text{ where } p = Z_o / 2R_d \quad (8)$$

$$\frac{k T_o B}{p} \left( \frac{2g}{1-g} \right) \left( 1 + p + \frac{p}{q} \right) \frac{f_s}{f_I} \quad (9)$$

From these expressions the noise figure for a single stage can be calculated as

$$F_1 = 1 + \frac{2}{q} \left[ \frac{1-g(1-g)}{1+g(1-g)} \right] + \frac{2g}{1+g(1-g)} \left[ \frac{1}{p} + \frac{f_s}{f_I} \left( 1 + \frac{1}{p} + \frac{1}{q} \right) \right] \quad (10)$$

Since the Hybrid Paramp would be operated at moderate rather than high gain, the noise measure,  $M$ , is useful and is calculated as

$$M = \frac{2}{q} \left[ \frac{1-g(1-g)}{g(2-g)} \right] + \frac{2}{(2-g)} \left[ \frac{1}{p} + \frac{f_s}{f_I} \left( 1 + \frac{1}{p} + \frac{1}{q} \right) \right] \quad (11)$$

Both  $M$  and  $F$  degenerate to the form for the conventional resonant Paramp if there is no loss associated with the inductance (since  $1/q$  is then zero). Of particular interest is the fact that at VHF frequencies the noise figure of the Hybrid Paramp should be lower than that of the resonant Paramp since the inductance provided is smaller and since at VHF frequencies inductance rather than varactor loss predominates.

#### A Practical Hybrid Paramp

An experimental Hybrid Paramp has been constructed at a signal frequency of 30 MHz with an idling frequency of around 800 MHz. This signal frequency was selected in preference to a microwave frequency in order to confirm the noise figure advantage of the Paramp.

The varactor diode selected was AEI DC 4312B/1 with a zero bias capacitance of 28 pF, and a capacitance of 11.5 pF at -4 volts bias. It is encapsulated in an S4 package.  $R_d$  is 0.7 Ω.

A gain of 6 dB was selected as the design value - this corresponds to a  $g$  of about 0.7, which requires a signal inductance of 5 μH.

The calculated  $Z_o$  for the T-section is 317 Ω.

The idler frequency was trimmed by means of a variable air spaced 4 pF capacitor in series with the idler inductance. Two parallel filters were provided in the series arms of the T-section and rejected the pump and idler frequencies. The photograph (Fig. 4) below shows the layout of the components.

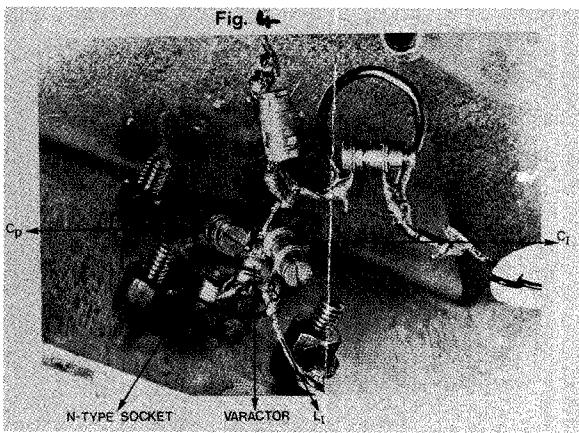


Fig. 5 shows the theoretical and measured variation of gain with pump power - showing the predicted result; in particular the slope is zero at the gain peak. At 6 dB gain the measured 3 dB bandwidth was 8 MHz.

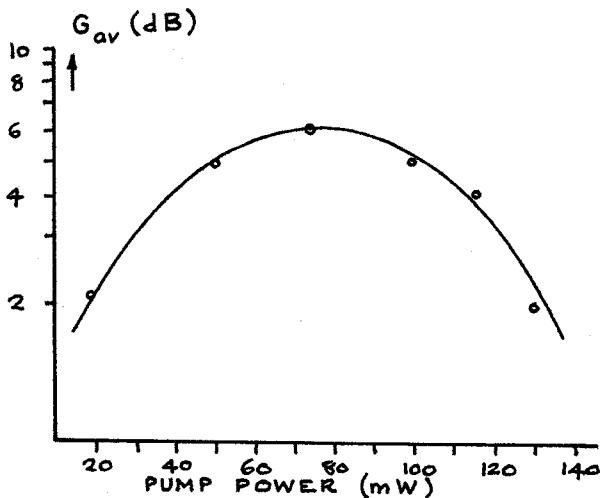


Fig. 5 Observed variation of gain with pump power

The noise figure of the Paramp was measured by following the Paramp with a variable calibrated attenuator and a transistor amplifier. The overall noise figure was measured with a shot noise diode and plotted against attenuation, as shown in Fig. 6 below. The slope of this straight line is  $(F_1 - 1/G_{av1})$  where the subscript 1 refers to the Paramp. This showed that the noise temperature was  $18^{\circ}\text{K}$  compared with a calculated value of  $17^{\circ}\text{K}$  at 6 dB.

This noise performance was confirmed by constructing a second identical amplifier and measuring the overall noise figure of the two amplifiers in cascade using the same technique. The measured value is  $22^{\circ}\text{K}$  which is close to the theoretical value of  $21^{\circ}\text{K}$ . The noise measure calculated from these measured results is 0.082.

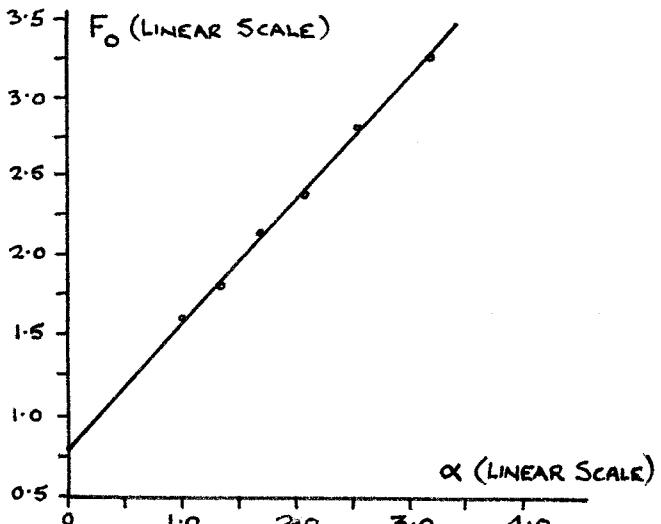


Fig. 6 Plot of overall noise figure  $F_o$  against  $\alpha$

#### Conclusions

It is concluded that the Hybrid Paramp has useful properties and that gain and noise figures are in accordance with theory. In particular the need for high pump power stability is removed since the gain is independent of pump power at the maximum design gain. The Hybrid Paramp also gives a superior noise performance at frequencies at which the inductor loss is significant in comparison with the capacitor loss.

A Hybrid Paramp with a noise temperature of  $18^{\circ}\text{K}$  and 6 dB has been successfully constructed at 30 MHz.

#### References

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3. WONG, A.S., and AITCHISON, C.S.: 'Hybrid parametric amplifier at 30 MHz with a noise temperature of  $18^{\circ}\text{K}$ ', ibid., 1976, 12, pp 687-688